INTRO TO COMPUTER SYMBOLIC MATH PACKAGES—AND HWK

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1. WHAT DO THEY DO?

Symbolic algebra packages work with whole eequations involving variables and unknown constants, instead of just numbers. The sorts of things they do include

- Expanding expressions (e.g., multiplying out products, collecting terms, trig identities), factoring polynomials, partial fractions decompositions—generally any sort of symbolic rearrangment.
- Solving equations, such as finding the roots of polynomials or solving k equations involving k unknowns.
- Differentiating and integrating functions, even computing limits.
- Solving differential equations (next part of the course)

Their power comes from having infinite patience to applying specific rules and patterns in sequence, and from having a large "knowledge base" of formulas and techniques programmed in. In theory, anything they can do could also be done by a human, but it might be so ugly that no human would ever actually do it.

The packages also are generally able to do numeric calculations to arbitrary precision (e.g. compute $\tan\pi/7$ to 100 decimal places) and draw fancy 2D and 3D graphics. These features are not actually symbolic math, but are often useful adjuncts to it.

The greatest power lies in combining several of these capabilities. For instance, you can take a real-life "messy" integral or differential equation, solve it, modify it into a slightly different function, differentiate it and find the critical points, and finally plot the graph, all in just a few minutes.

1.1. **Limitations.**

First, the packages are necessarily quite complex. It's easy to forget how vague we can be in posing a problem. For instance, consider the question "What is $\int_0^2 (Ax-c)^r\,dx$?" It looks easy—just use power rule and plug in the endpoints. But what if $r=-1$ or $A=0$? A computer package can blithely make assumptions to bar such special cases, but it's hard for it to second-guess us and the assumptions can be disastrously wrong in a following step. Alternatively, the computer can give a complicated answer which deals with all possible cases, but this can be horribly unwieldy. There is no easy solution, though the packages are getting "smarter" at guessing what the user means. However, input syntax and output format can still be quite picky and counterintuitive. *

Second, the "knowledge base" is only as big as it has been programmed. Computations which depend on some sneaky trick will only succeed if some schmuck programmed it in. The packages are not "intelligent".

Third—this sounds silly but is important—the packages cannot do what is impossible. There are many integrals which just have no answer in terms of elementary functions. There is no algorithm to systematically factor polynomials of degree >4. For many such questions, there is much *partial* information known about the "answer", but this is not something that can be done automatically.

2. WHAT PACKAGES ARE AVAILABLE?

Nowadays, all of these are available for PCs, Macs, and UNIX boxes. It used to be that you needed a particularly fast machine, but the high-powered graphics and audio processing in modern computer games actually needs more horsepower than most symbolic math.

2.1. **Mathematica.**

(`www.mathematica.com`) is the oldest, most common, and arguably most powerful and also most daunting, of the packages. You can buy a student version for under $100, or pay something like $750 for a full professional version including the full knowledge base.

It's installed on the 6 SGI workstations at USITE. Log on with your harper userid and choose Mathematica from the Apps menu. Drag the symbol palette out of the way and click in the document window (you won't see a cursor but that's fine) and type away. To have Mathematica evaluate a command you typed, you have to press Enter on the numeric keypad, not the Return key. Arguments to functions and commands should be in square brackets, and names of commands and internal functions all start with a capital letter. H

Date: April 9, 1999.

2.2. **Maple.** (www.maplesoft.com) does almost all Mathematica does, but feels a bit more like you're computer programming and doesn't generate as fancy output. I find it less daunting, but maybe that's because I know it better. Similar pricing to Mathematica, I think.

H It's installed on all the Macs at USITE.You have to end every line of command input with a semicolon, and put a $*$ between numbers or variables that are to be multiplied.

The Maple "engine" is also used in MathCad (www.mathcad.com, I assume), a program that actually emphasizes graphing and numerical calculations. Also under $100 for students, and maybe a better mixture of features for those of you who will need to write lab reports. Not in the public computer labs.

2.3. **MuPAD.** (www.mupad.de) is a new program being developed in Germany. Last time I checked, it had a slightly strange user interface, and sometimes took a simple problem and gave a complicated, though correct answer. However, you can download it for FREE (Windows or Mac), even try it online (go to the demos section).

2.4. **The Integrator.** (www.integrals.com) is not a piece of software, but rather a web page which will do INTEGRALS
! FOR FREE. It's based on Mathematica (and actually run by them for publicity). Good online help. Give it a try—square brackets and capitalized function names like Mathematica.

3. Homework

Due Friday April 30 into my box. You will need to use one of the packages mentioned above. You can work with a partner[1]—probably a good idea, especially if you are the sort who only grudgingly gets along with computers. Feel free to ask me for help, and let me know if you have trouble. This is the first time I'm giving this sort of assignment, and I don't really know how challenging people will find it to work with unfamiliar software. Besides, who knows what can crash at USITE!

I'm giving you a lot of time in case you have trouble, USITE is busy, or whatever. However, I strongly suggest you do it early. Some of the math modelling homework in coming weeks will involve integrals which *can* be done reasonably my hand, but will be less annoying by computer (e.g. The Integrator).

Don't use computer programs to do the homework assigned before today!

Question A. Choose any 5 of the 10 integrals below and have the computer solve them. Either copy your answer by hand off the screen, or print out your whole session on paper and hand that in.

For each of the 5 integrals, write one sentence to explain what techniques you think the computer used to solve the integral, e.g. partial fractions, messy u-substitution, or double angle trig identities, ...

Warning: The computer will give a less than ideal answer in some of these. That's fine; just make note of it and explain what happened. [In one case, there is no answer in terms of elementary functions. In another, the computer will express the answer in terms of a special function (whose name sounds like a dog barking) ... use the help system to figure out the definition of this function and tell it to me. In yet another, the computer will be unable to factor a polynomial and will only give a roundabout anwer. Finally, in one case the computer (or at least Maple, don't know about the others) will give an answer which actually makes no sense since the question is fatally flawed.]

$$\int (\sin x)^3 \cos(3x+2)\,dx \qquad \int \frac{x\,dx}{(3x^2+4x+5)^3} \qquad \int e^{1-x-x^2}\,dx$$

$$\int \frac{dx}{kx(N-x)} \qquad (k,N \text{ some constants. Careful you don't type } \frac{1}{k}x(N-x))$$

$$\int \frac{dx}{-gx+Cx^p} \qquad \int \frac{\sqrt{x}}{1+e^x} \qquad \int x(1+x)^{1/3}\,dx$$

$$\int \ln(-x^2)\,dx \qquad \int x^9 \sin x\,dx \qquad \int (x^7-1)^{-1}dx$$

(e^u is called Exp[u] in Mathematica and exp(u) in Maple. *ln* is called Log in Mathematica)

Question B. (no computer needed) For the integral marked $\boxed{*}$ above, try to write down a complete solution without making any assumptions on the constants r, A, and c except that they are real numbers. Your answer will have several special cases as well as a "generic" case when these exceptions do not occur.

[1] This means both of you at the computer together, not one doing the work and the other co-signing it.