INTEGRATION BY COMPUTER WITH MAPLE   Calculus 153/22   M. Pergler, April 99

Suppose we want to integrate the following monster rational function.

```
> f :=(2*x^4-8*x^3-20*x^2+216*x-286)/(x^5-11*x^4+58*x^3-134*x^2+21*x+225);
```

$$f := \frac{2x^4 - 8x^3 - 20x^2 + 216x - 286}{x^5 - 11x^4 + 58x^3 - 134x^2 + 21x + 225}$$

Maple echoed back what I typed in "prettyprinted" form so I could see I didn't misplace ( )'s or something. The notation f := just means "Define f to mean..."

Let's integrate f(x). Maple takes about 2.5 seconds to do all the work on my home computer.

```
> int(f,x);
```

$$-\ln(x+1) - 2\frac{1}{x-3} + \ln(x-3) + \ln\left(x^2 - 6x + 25\right) + \frac{1}{2}\arctan\left(\frac{1}{4}x - \frac{3}{4}\right)$$

Maple is excellent at all sorts of symbolic manipulation, for instance partial fractions.

```
> convert(f,parfrac,x);
```

$$-\frac{1}{x+1} + 2\frac{1}{(x-3)^2} + \frac{1}{x-3} + 2\frac{-2+x}{x^2-6x+25}$$

It actually took no time at all for Maple to compute this...this is because it already had to do it when integrating f(x) and still had the answer in memory.

Of course, no computer package can solve an integral that doesn't have a closed form answer.

```
> g:=(sin(x)+tan(sqrt(x)))*(1+exp(1/x));
```

$$g := \left(\sin(x) + \tan\left(\sqrt{x}\right)\right)\left(1 + e^{\frac{1}{x}}\right)$$

```
> int(g,x=1..2);
```

$$\int_1^2 \left(\sin(x) + \tan\left(\sqrt{x}\right)\right)\left(1 + e^{\frac{1}{x}}\right) dx$$

Maple tried all the integration rules in its database (took about 12 seconds) and couldn't figure out a thing. So it just returns the same formula we gave it, in case we want to try something else. For instance, we can ask it to numerically integrate instead, to 50 digit precision. (Since Maple couldn't find an antiderivative, it will just use the thin rectange under the curve method.)

```
> evalf(",50);
```

$$11.968469205125829593528381985704308336376158733789$$

[The " means to use the last answer. Newer versions of Maple, as well as other programs, use % instead of "]

What Maple really does is very persistent pattern matching...it is not "artificially intelligent", and sometimes it gives an answer which gives little new information. Note I just stuck in the expression without defining it as a function first.

```
> int(sin(x^2),x);
```

$$\frac{1}{2}\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)$$

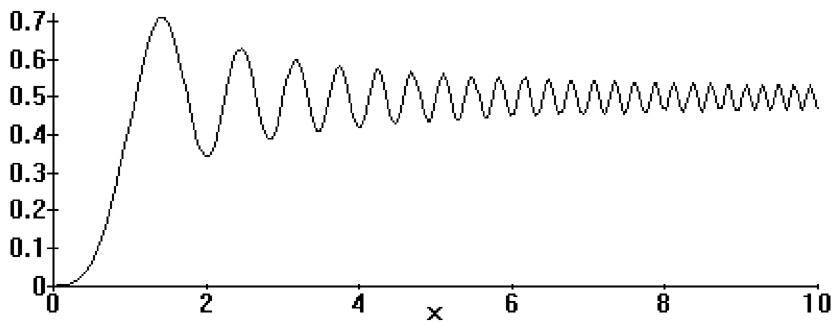
What in the world is FresnelS( ). Let's ask for help.

```
> ?FresnelS
```

========(edited reply from help window)=======
FUNCTION: FresnelS - The Fresnel Sine Integral
- The Fresnel sine integral is defined as follows: FresnelS(x) = int(sin(Pi/2*t^2), t=0..x);
======================================

This is essentially exactly what we asked it to do! So the integral we asked for does not have a "nice" answer, but it is sufficiently important that the function it defines is given a name. Maple just did a trivial substitution before giving us the answer back.
[It so happens that this integral comes up in the theory of Fresnel diffraction (optics), hence the name. You may see it in a physics course, and it showed up on the B part of one of my tests last quarter.]

```
> plot(FresnelS(x),x=0..10);
```



Finally, from the "Oooh! Aaaah!" department. This should be in color, but for some reason the PDF file isn't.

```
> with(plots):tubeplot({[cos(t),sin(t),0,t=Pi..2*Pi,numpoints=15,radius=0.25*(t-Pi)],
> [0,cos(t)-1,sin(t),t=0..2*Pi,numpoints=45,radius=0.25]});
```