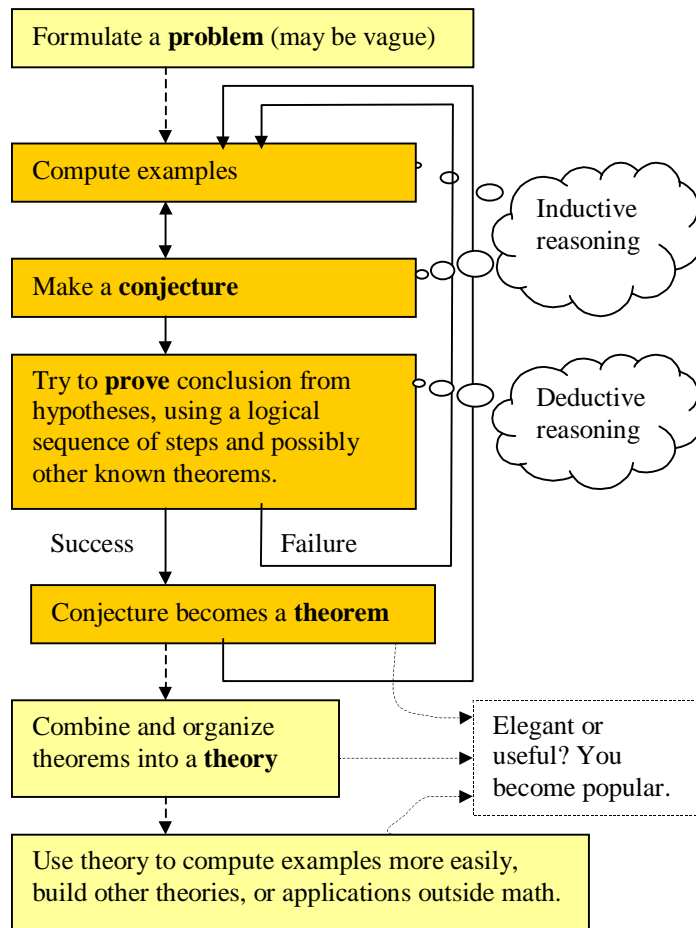# How does a mathematician think? Martin Pergler, 99.09.23, for Calc 151/30

Simplistic example:
What happens when you do arithmetic with rational numbers?

$4 \times 5 = 20$. $-1/6 \times 2/3 = -1/9$

Hmm. Seems you get rationals.

Conj. If x and y are rational, so is xy

Proof. Write $x=p/q$ and $y=r/s$ with p,q,r,s integers, possible since x and y are rational. Then $xy=pr/qs$. Since a product of integers is an integer, this is a ratio of two integers, hence rational.

Theorem. ...

Try same with +, -, and /. Careful with /0.

The rational numbers are "closed under arithmetic operations".

Build calculators? Approx. irrat. by rat.? etc.

Flowchart:
- Formulate a **problem** (may be vague)
- Compute examples
- Make a **conjecture**
- Try to **prove** conclusion from hypotheses, using a logical sequence of steps and possibly other known theorems.
- Success / Failure
- Conjecture becomes a **theorem**
- Combine and organize theorems into a **theory**
- Elegant or useful? You become popular.
- Use theory to compute examples more easily, build other theories, or applications outside math.

Inductive reasoning

Deductive reasoning

Notes:
1. Sometimes theorems are called propositions, lemmas, or corollaries. These words have specific connotations which we will discuss as they arise.
2. Remember: conjecture=suspected to be true, usually based on examples, but not proven to be true in every case. Theorem=proven to be the case from hypotheses, not "by example"
3. Proofs can use results of theorems already proven. This is part of the art of formulating a theory.
4. In science, theory=not necessarily proven, i.e. "only a theory", but internally consistent. In math, a theory generally consists of proven theorems (terminology varies a bit)
5. The proof in the example should include checking denominator is not zero. Omitted for lack of space.

Who does what?	150s lectures	150s homework	"Professional mathematician"	Person using mathematics
Learning about a developed theory	Lots	Not in hwk, but read in text on your own	Some	Some
Forming conjectures (inductive reasoning)	Some	Some, with guidance	Lots	Some
Proving theorems (deductive reasoning)	Often	A fair bit	Lots	Very little
Computing examples using theory (plug'n'chug)	Some	Often	Very little, except to formulate conjectures	Lots